The unrestricted latent class model

Manifest (observed) variables $A, B, C,$ and D having respectively $I, J, K,$ and L classes. Prob that an individual (ind) is at level i, j, k, l is π_{ijkl}.

$$\pi_{ijkl} = \sum_{t=1}^{T} \pi_{ijklt}^{ABCDX} \tag{1}$$

In words, inds can be divided into non-overlapping, mutually exclusive latent classes.

$$\pi_{ijklt}^{ABCDX} = \pi_t^X \pi_{it}^{\bar{A}X} \pi_{jt}^{\bar{B}X} \pi_{kt}^{\bar{C}X} \pi_{lt}^{\bar{D}X} \tag{2}$$

In words, the manifest (observed) vars are independent given the latent class. Note that $\pi_{it}^{\bar{A}X}$ is prob that an ind is at level i with respect to (wrt) var A given that she is at t wrt X (i.e. wrt her latent class).

The following conditions are required:

$$\sum_{t=1}^{T} \pi_t^X = 1, \sum_{i=1}^{I} \pi_{it}^{\bar{A}X} = 1, \sum_{j=1}^{J} \pi_{jt}^{\bar{B}X} = 1, \sum_{k=1}^{K} \pi_{kt}^{\bar{C}X} = 1, \sum_{l=1}^{L} \pi_{lt}^{\bar{D}X} = 1 \tag{3}$$

$$\pi_t^X = \sum_{i,j,k,l} \pi_{ijklt}^{ABCDX}, \text{and} \tag{4}$$

$$\pi_t^X \pi_{it}^{\bar{A}X} = \sum_{j,k,l} \pi_{ijklt}^{ABCDX}. \tag{5}$$

And eq'ns for $B, C,$ and D corresponding to (5).

I find this counter intuitive, but for the prob that an ind is in latent class t given that she is at (i, j, k, l), Goodman uses $\pi_{ijklt}^{ABCD\bar{X}}$:

$$\pi_{ijklt}^{ABCD\bar{X}} = \pi_{ijklt}^{ABCDX} / \pi_{ijkl}^{ABCD} \tag{6}$$

The last eq'ns that Goodman needs prior to describing the algorithm just rewrite (4) and (5) using (6):

$$\pi_t^X = \sum_{i,j,k,l} \pi_{ijkl}^{ABCD} \pi_{ijklt}^{ABCD\bar{X}} \tag{7}$$

$$\pi_{it}^{\bar{A}X} = \left(\sum_{j,k,l} \pi_{ijkl} \pi_{ijklt}^{ABCD\bar{X}} \right) / \pi_t^X \tag{8}$$

Now we give the eq'ns governing the convergence algorithm. These are given in 2 sets of eq'ns, those in the 2nd set depending on those from the first. Let p_{ijkl} be the proportion of obs seen to fall into cell i, j, k, l. Then, the first set is:

$$\hat{\pi}_{ijkl} = \sum_{t=1}^{T} \hat{\pi}_{ijklt}^{ABCDX} \tag{9}$$

$$\hat{\pi}_{ijklt}^{ABCDX} = \hat{\pi}_t^X \hat{\pi}_{it}^{\bar{A}X} \hat{\pi}_{jt}^{\bar{B}X} \hat{\pi}_{kt}^{\bar{C}X} \hat{\pi}_{lt}^{\bar{D}X} \tag{10}$$

$$\hat{\pi}_{ijklt}^{ABCD\bar{X}} = \hat{\pi}_{ijklt}^{ABCDX} / \hat{\pi}_{ijkl} \tag{11}$$

The second set of eq'ns is:

$$\hat{\pi}_t^X = \sum_{i,j,k,l} p_{ijkl} \hat{\pi}_{ijklt}^{ABCD\bar{X}} \tag{12}$$

$$\hat{\pi}_{it}^{\bar{A}X} = \left(\sum_{j,k,l} p_{ijkl} \hat{\pi}_{ijklt}^{ABCD\bar{X}} \right) / \hat{\pi}_t^X \tag{13a}$$

$$\hat{\pi}_{jt}^{\bar{B}X} = \left(\sum_{i,k,l} p_{ijkl} \hat{\pi}_{ijklt}^{ABCD\bar{X}} \right) / \hat{\pi}_t^X \tag{13b}$$

$$\hat{\pi}_{kt}^{\bar{C}X} = \left(\sum_{i,j,l} p_{ijkl} \hat{\pi}_{ijklt}^{ABCD\bar{X}} \right) / \hat{\pi}_t^X \tag{13c}$$

$$\hat{\pi}_{lt}^{\bar{D}X} = \left(\sum_{i,j,k} p_{ijkl} \hat{\pi}_{ijklt}^{ABCD\bar{X}} \right) / \hat{\pi}_t^X \tag{13d}$$

Now we give the algorithm:
Start with a reasonable (well, all entries on the open interval (0, 1) for starters) vector $\pi(0) = \{\pi_t^X(0),\ \ \pi_t^{\bar{A}X}(0),\ \ \pi_t^{\bar{B}X}(0),\ \ \pi_t^{\bar{C}X}(0),\ \ \pi_t^{\bar{D}X}(0)\}$ and use these and successively generated values to solve the left hand sides (lhs) of eq'ns (10), (9), and (11). Then use these new values along with the observed p_{ijkl} to solve first eq'n (12) then (13a)-(13d). Next, use the values obtained from (12) and (13) to form $\pi(1)$ and so on until convergence.

That's pretty much it.

Note a couple of things:

- These methods are apparently full of cases where many local optima are found. Compute the likelihood ration chi-square for each set of values to find which set of values minimize it.
- A word of caution: One should probably read Goodman's prescribed methods of determining if the model is identifiable before one spends a great deal of time trying to determine which of a set of solutions is optimal.